MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2022/2023 (TERM ID 2210 & TERM ID 2215)

EEM1016 – ENGINEERING MATHEMATICS I

(RE/ME)

13 FEBRUARY 2023
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of four pages including cover page with five questions only.

2. Attempt **ALL** questions. The distribution of the marks for each question is given.

3. Please write down all your answers in the answer Booklet provided. Show all relevant steps to obtain maximum marks.

Question 1

a) Evaluate the limit

$$\lim_{x\to 1}\frac{\sin(\pi x)+\pi x-\pi}{(x-1)^3}$$

[4 marks]

b) Given the function $f(x) = 2x^3 - 21x^2 + 60x + 1$ for the interval [0,5].

(i) Find and classify all the local extrema of $f(x)$ using the Second Derivative Test. Hence, calculate the local extreme values.

(ii) Find the absolute extreme values.

[10+2 marks]

c) Evaluate

$$\int x^{\frac{1}{2}}\left(x^{\frac{3}{2}}+1\right)^{\frac{1}{2}}dx$$

[4 marks]

Question 2

a) Find the sum of the following infinite series.

$$\sum_{n=1}^{\infty}\frac{2^n-3^n}{7^n}$$

[5 marks]

b) Determine the radius and interval of convergence for the following Taylor series. You are not required to test the endpoints for convergence.

$$\sum_{n=1}^{\infty}\frac{n!}{n+1}(x-2)^n$$

[8 marks]

c) The Maclaurin series of $f(x)$ is given by

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n$$

Suppose that

$$f(x)=e^{2x}$$

(i) Find $f(0)$, $f'(0)$, $f''(0)$, and $f'''(0)$. Hence, find an expression for $f^{(n)}(0)$.

(ii) Write down the Maclaurin series of $f(x)$ in summation notation as well as expansion up to the fourth term.

[Note: You are not required to find the interval of convergence.]

[4+3 marks]

Continued...

Question 3

a) Using implicit differentiation, find

$$\frac{\partial z}{\partial y}$$

given that

$$x^2 \sin(y) + y^2 \sin(z) + \cos(xyz) = 0$$

[3 marks]

b) Find the equation of the tangent plane to the surface of

$$f(x,y) = x^2 + \ln y - \frac{\sin(\pi xy)}{\pi}$$

at $(x_o, y_o) = (2,1)$.

[6.5 marks]

c) Find the local extreme value of

$$f(x,y) = 2x^2 + y + 2xy$$

subject to $2x + 3y = 2$ by using Lagrange multipliers. Is the local extreme value a local minimum or local maximum? Explain.

[10.5 marks]

Question 4

a) Given a complex number $w = -1 + i2$, find

i) the exponential form of w.

ii) all the roots for $z^2 = w$. Leave the answers in polar form.

[5+5 marks]

b) Suppose a plane contains three points A(2,3,0), B(2,0,–1) and C(4,–1,7).

i) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Hence, find a normal vector to the plane.

ii) Find the equation of the plane. Leave the plane equation in the form $ax + by + cz = d$.

iii) Find the parametric equations of the line passing through point C and parallel to the normal vector found in part i).

[5+2+3 marks]

Continued...

Question 5

Suppose a periodic function is given by

$$f(t) = t, \text{for } -1 < t < 1,$$
$$f(t) = f(t+2)$$

a) Sketch the waveform of $f(t)$ for $-3 < t < 3$.

[3 marks]

b) Label in the waveform of part a), the interval (with its value) of one fundamental period T. Hence, calculate the angular frequency ω.

[3 marks]

c) Is the periodic function an odd or even function?

[1 mark]

d) Show that the Fourier series is

$$f(t) = \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n\pi} \sin(n\pi t)$$

[9 marks]

e) Determine the value of Fourier series at $t = 0.5$. Hence, show that

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$$

[4 marks]

End of Paper